Begin a supportive year-long relationship of math learning
Understand how a problem-based lesson looks and feels

ALGEBRA 2 LESSON PLAN

Family Math Night

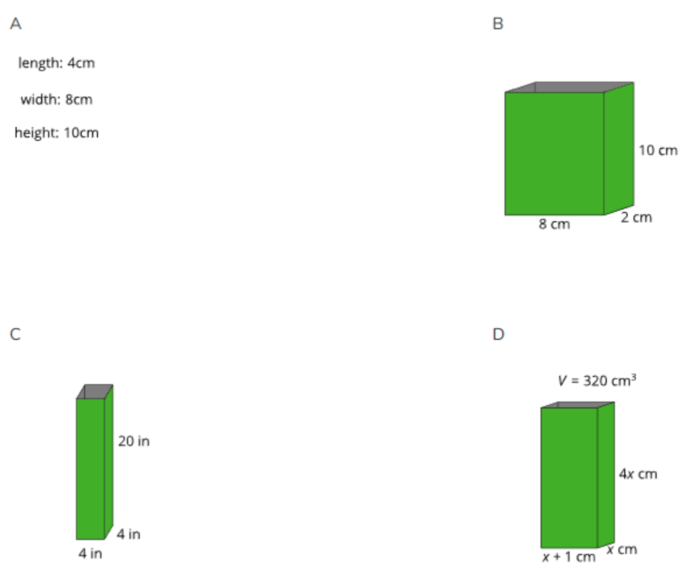
HANDOUT 1: Family Night Lesson Plan
HANDOUT 2: Warm-up and Activity
HANDOUT 3: Family Materials
HANDOUT 4: Condensed Lesson Plan

Illustrative® Mathematics
LEARN MATH FOR LIFE

Learning Goals:

- Begin a supportive year-long relationship of math learning.
- Understand how a problem-based lesson looks and feels.
- Understand the connection between what students are learning in class and the family support materials.
- Understand how to access the student and family materials.

Algebra 2 Lesson Plan, Unit 2, Lesson 1

Time	Activity	Teacher Notes
10 minutes	**Framing** What does it mean to you to “do math”?	• Ask participants to write down 2–3 words or phrases they would use to describe what it means to do math. This could be based on their own experiences. • Ask if anyone would like to share. Record and display their words or phrases for all to see. • Tell them after we do the activities, we will revisit this list.
10 minutes Handout 2	**Warm-up: Which Three Go Together: Boxes** Which three go together? Why do they go together? 	• Tell participants this is the first lesson in the second unit. • Give each participant a copy of the handout. • Tell families that this warm-up helps elicit what understanding students currently have for determining the volume of a figure, which is the main context for the lesson. • Do this with families as stated in the curriculum. After some quiet think time, try to elicit ideas for all four figures.

20 minutes

Handout 2

Activity: Building Boxes

Your teacher will give you some supplies to construct an open-top box.

1. Cut out a square from each corner of a sheet of paper, and then fold up the sides.
2. Calculate the volume of your box, and complete the table with your information.

side length of square cutout (in)	length (in)	width (in)	height (in)	volume of box (in^3)
1				

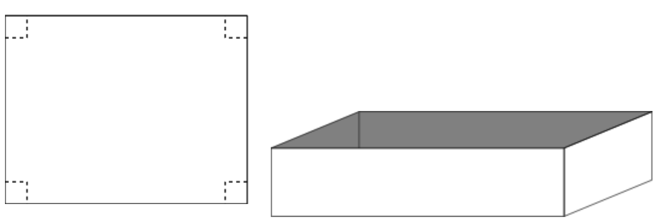
- Distribute sheets of 8.5 inch by 11 inch paper, scissors, and rulers.
- Launch this activity following the directions in the curriculum.
- Give participants time to work independently or in small groups to complete a box, fill in a table row, and plot their point on the displayed axes.
- Once groups have completed, select groups to share how they calculated the volume of their box either by measuring directly or by working out what the edge lengths would be based on the size of the cutout.
- If time allows, invite participants to consider how to calculate the volume of a box if the side length of the square cutout is x. Otherwise, share how students would think through the problem to end at $V(x) = (11 - 2x)(8.5 - 2x)(x)$. Tell participants that this is one example of a polynomial equation.

10 minutes

Handout 3

Family Materials

Unit 1, Lesson 2 Student Lesson Summary
A box can be created by removing squares from each corner of a rectangle of paper.



Let $V(x)$ be the volume of the box in cubic inches, where x is the side length, in inches, of each square removed from the four corners.

To define $V(x)$ using an expression. . .

- Tell participants that each lesson has a summary which highlights key ideas, vocabulary, and often worked examples.
- Give each participant a copy of the student lesson summary for this lesson and the family materials for the unit.
- Tell them as they read to underline ideas they touched on in the warm-up and activity today.
- Give them time to read the lesson summary.

	Unit 2 Family Materials In this unit, your student will learn about a kind of function, called a *polynomial*. A polynomial is a sum of terms involving only one letter, called a variable, and in which the exponents of the variable are whole numbers. For example, $3x^2 - x^2 + 10$ and $5x^6$ are polynomials. But $6x^{-2}+2x^{-1}$ is not, because. . .	• Explain the goals of the family materials, and ask families to read through them and try the task with their students in the next few days. • Tell them this is the format used for the family materials for every unit. • Pull up the curriculum website at AccessIM.org and show families how to access the family and student materials. • Take them on a tour of the lesson they worked with.
10 minutes	**Closing** What might it mean for students to "do math"?	• Revisit the earlier question. • Ask participants, "Based on this experience, how might students describe doing math this year?" • Highlight any differences from earlier words given during the launch of the lesson.
Modifications	**45 minute session:** Consider reducing the warm-up, activity, and closing sections by 5 minutes each. This can be done by pre-plotting the data from the table instead of having groups plot their own points and not revisiting the earlier words from the launch during the close. We suggest you do not shorten the time for the family support materials overview. **30 minute session:** Consider cutting the warm-up altogether and reducing the launch, activity, family support materials, and closing sections by 5 minutes each. **More than 1 hour:** Consider giving participants more time to explore the curriculum resources and brainstorm a list of ways these resources could help them support their child at home.	

UNIT 2, LESSON 1

Let's Make a Box

Algebra 2

1.1 Warm-up

Which Three Go Together: Boxes

Which three go together? Why do they go together?

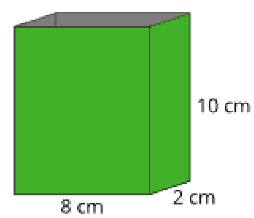
A

length: 4cm

width: 8cm

height: 10cm

B



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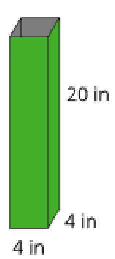

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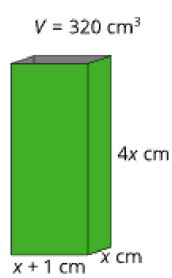


1.2 Activity

Building Boxes

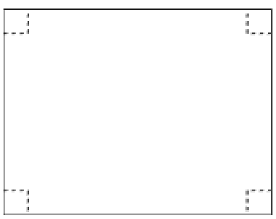
Your teacher will give you some supplies to construct an open-top box.

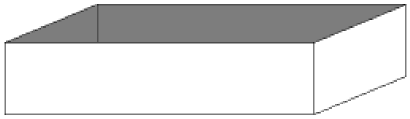
1. Cut out a square from each corner of a sheet of paper, and then fold up the sides.
2. Calculate the volume of your box, and complete the table with your information.

side length of square cutout (in)	length (in)	width (in)	height (in)	volume of box (in^3)
1				

Student Lesson Summary

A box can be created by removing squares from each corner of a rectangle of paper.





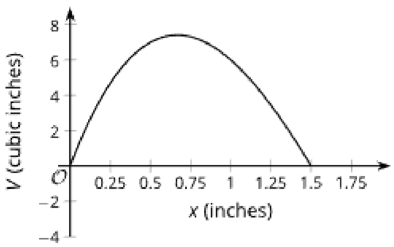
Let $V(x)$ be the volume of the box in cubic inches, where x is the side length, in inches, of each square removed from the four corners.

To define V using an expression, we can use the fact that the volume of a cube is $(length)(width)(height)$. If the piece of paper we start with is 3 inches by 8 inches, then:

$V(x) = (3 - 2x)(8 - 2x)(x)$

What are some reasonable values for x? Cutting out squares with side lengths less than 0 inches doesn't make sense, and similarly, we can't cut out squares larger than 1.5 inches, since the short side of the paper is only 3 inches (since $3 - 1.5 \cdot 2 = 0$). You may remember that the name for the set of all the input values that make sense to use with a function is the domain. Here, a reasonable domain is somewhere larger than 0 inches but less than 1.5 inches, depending on how well we can cut and fold!

By graphing this function, it is possible to find the maximum value within a specific domain. Here is a graph of $y = V(x)$ for x values between 0 and 1.5. It looks like the largest volume we can get for a box made this way from a 3-inch by 8-inch piece of paper is about 7.4 in^3.



UNIT 2

Polynomial Functions

Algebra 2

In this unit, your student will learn about a kind of function called a *polynomial*. A polynomial is a sum of terms involving only one letter, called a variable, and in which the exponents of the variable are whole numbers. For example, $3x^3 - x^2 + 10$ and $5x^6$ are polynomials. But $6x^2 + 2x^{-1}$ is not, because one of the exponents is negative. And $2xy - 7x$ is not, because it involves more than one variable. Your student will represent polynomial functions with graphs and equations, as they did in a previous course when they worked with two special kinds of polynomials—linear and quadratic functions.

Multiplication and division of numbers will be extended to polynomials, so this is a good time to refresh skills with multiplying and dividing numbers by hand. When numbers are multiplied, we often use the distributive property, in which each piece of one number is multiplied by each piece of the other number. For example, 34 is 30 plus 4, or 3 tens plus 4 ones. The tens and ones of each number are multiplied by the tens and ones of the other, and then all the results are added. When polynomials are multiplied, we also use the distributive property. Here is an example of each:

$$\begin{aligned}&(30+4)(10+5)\\&=30(10+5)+4(10+5)\\&=30\cdot 10+30\cdot 5+4\cdot 10+4\cdot 5\\&=300+150+40+20\\&=510\end{aligned}$$

$$\begin{aligned}&(x-7)(2x+3)\\&=x(2x+3)+(-7)(2x+3)\\&=x\cdot 2x+x\cdot 3+(-7)\cdot 2x+(-7)\cdot 3\\&=2x^2+3x-14x-21\\&=2x^2-11x-21\end{aligned}$$

Multiplication, with numbers or polynomials, can be represented in a lot of ways, and your student should find a way that makes sense and is useful. Ask your student to show you how to multiply polynomials.

Long division with polynomials looks a lot like long division with numbers. Here is an example of each:

$$\begin{array}{r}31\\12\overline{)372}\\\underline{-36}\\12\\\underline{-12}\\0\end{array}$$

$$\begin{array}{r}3x+1\\x+2\overline{)3x^2+7x+2}\\\underline{-3x^2-6x}\\x+2\\\underline{-x-2}\\0\end{array}$$

Division can also be represented in many ways, so if you or your student learned a different way of doing long division, that way can also be extended to polynomials.

Here are some tasks to try with your student:

1. Multiply 47 by 25, using any method you like. Try using that same method to multiply $(4x+7)(2x+5)$. What was the same? What was different?
2. Divide 372 by 12, using any method you like. Then represent the division another way—for example, using pictures or words.
3. Factor these expressions. Check your answers by multiplying the factors. When you were factoring and multiplying, how did you know what to do at each step?
   a. x^2+5x+6
   b. x^2+2x-8

Solution:

1. One way to multiply 47 by 25 is to use a standard multiplication algorithm. We can also do something similar with $(4x+7)(2x+5)$. Just as we multiplied 47 by 5 and then by 20 and then added the results together, we can multiply $4x+7$ by 5 and then by $2x$ and then add the results together. Here are two versions:

$$\begin{array}{r} 47 \\ \underline{\times 25} \\ 235 \\ \underline{+940} \\ 1175 \end{array} \qquad \begin{array}{r} 4x+7 \\ \underline{\times\ \ 2x+5} \\ 20x+35 \\ \underline{+\ \ 8x^2+14x+\ \ 0} \\ 8x^2+34x+35 \end{array}$$

2. One way to divide 372 by 12 is the standard division algorithm (shown earlier). Another way to do it is by subtraction. To be more efficient, we can take away groups of 120 (ten 12s) until the result is less than 120, and then take away groups of 12. We can take away 3 groups of 120 and 1 group of 12 from 372, and then we have nothing left over. So there are 31 groups of 12.
3. a. $x^2+5x+6=(x+3)(x+2)$
   b. $x^2+2x-8=(x+4)(x-2)$
   When factoring the first expression, I thought of two numbers whose product was 6 and sum was 5, and found 2 and 3. Similarly, for the second expression, the product of 4 and -2 is -8, and the sum of 4 and -2 is 2.

   When multiplying the factors to check my answer, I knew that I had to multiply each piece of the first factor by each piece of the second factor.

CONDENSED LESSON PLAN

Algebra 2, Unit 2, Lesson 1: Let's Make a Box

Lesson Goals

- Generalize from specific calculations to create a polynomial to model the volume of a constructed box.
- Interpret features of a graph of a polynomial.

Required Materials

- blank paper
- graphing technology
- rulers
- scissors
- tape

Lesson Narrative

In this lesson, students investigate the volumes of open-top boxes made from single sheets of paper. Students cut squares from each corner of a sheet of paper and fold the remaining paper to create an open-top box, which can serve as a visual reference while exploring polynomial functions further in this unit.

Students graph input-output pairs and interpret the points to estimate the side length that results in the box with the largest volume (MP2). The volume of the box is a function of the side length of the square cut from each corner, which affects all three dimensions of the box. This leads students to write a cubic expression for the volume in the form of length x width x height, providing students with a path into polynomial functions. Students also determine a reasonable domain for the function.

The focus of this lesson is to interpret features of a polynomial graph, even though students will not be introduced to the term polynomial until a future lesson. Students will learn more precise language to describe graphs and equations of polynomials later in this section.

1.1: Which Three Go Together: Boxes (5 minutes)

Activity Narrative

This warm-up prompts students to compare four open-top boxes with specific dimensions. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the boxes in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the information about the 4 boxes for all to see. Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three things that go together and can explain why. Next, tell students to share their response with their group and then to work together to find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

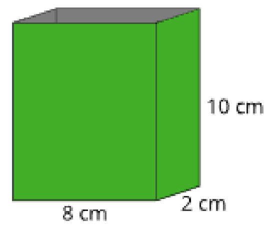
A

length: 4cm

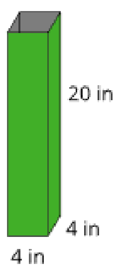
width: 8cm

height: 10cm

B



C



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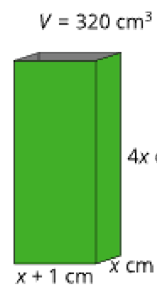


Student Response

Sample responses:

- A, B, and C go together because the dimensions are all given directly.
- A, B, and D go together because they are all measured in cm.
- A, C, and D go together because they all have a volume of 320 cubic units.
- B, C, and D go together because they all include an image of the box.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure that the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "cubic units," and to clarify their reasoning as needed. Consider asking:

- "How do you know _______?"
- "What do you mean by _______?"
- "Can you say that in another way?"

1.2: Building Boxes (20 minutes)

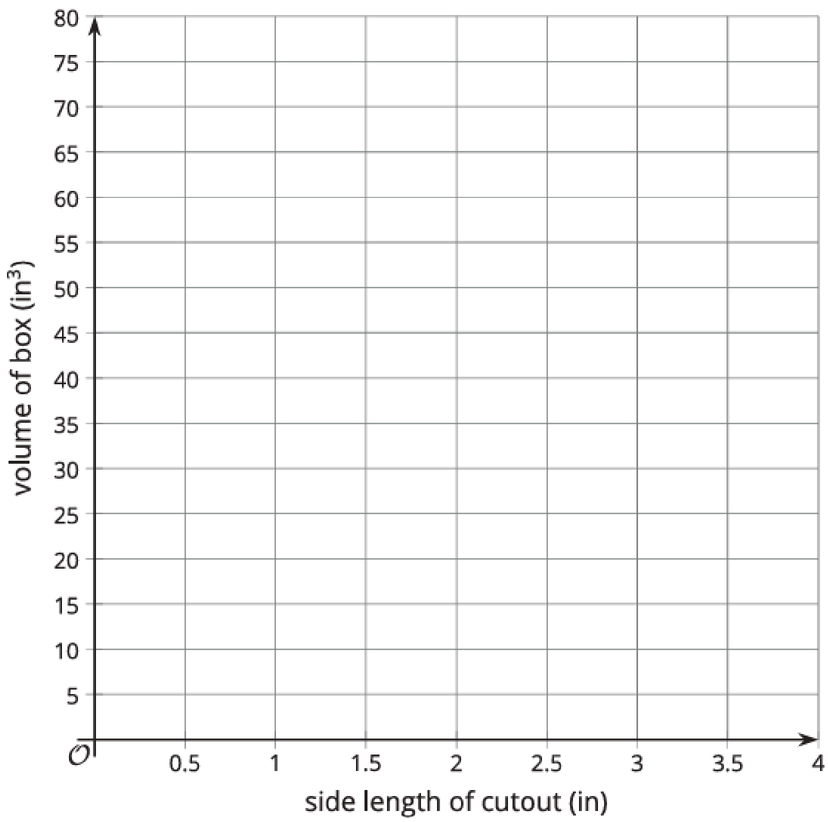
Activity Narrative

This activity offers a hands-on introduction to the mathematical work of modeling the volume of a box using a polynomial function. Students do not need to develop an equation with variables for the volume of the box or identify the greatest possible volume at this time, as that is the focus of the following activity.

Monitor for students using logical reasoning to figure out the volume of their boxes instead of, or in addition to, measuring directly to share during the whole-class discussion.

Launch

Arrange students in groups of 2. Display the table from the *Task Statement* for all to see, and a labeled set of axes like the one shown here.



Demonstrate how to construct a box from an 8.5-inch by 11-inch sheet of paper by cutting out identical squares, with a side length of 1 inch, from each corner. Then measure to fill in the length, width, and height columns of the table. Calculate the volume of the box and add a point, to the displayed axes, that represents the relationship between the side length of the cutout square and the volume of the box.

Assign each group a side length between 0.5 inch and 4 inches, in half-inch increments. Adjust these side lengths as needed if using different-sized paper. Tell groups to construct a box by cutting squares out of each corner using their assigned side length, and then to add a point to the graph representing their box.

Student Task Statement

Your teacher will give you some supplies to construct an open-top box.

1. Cut out a square from each corner of a sheet of paper, and then fold up the sides.
2. Calculate the volume of your box, and complete the table with your information.

side length of square cutout (in)	length (in)	width (in)	height (in)	volume of box (in^3)
1				

Student Response

Sample response based on a piece of paper with dimensions of 8.5 in x 11 in.

side length of square cutout (in)	length (in)	width (in)	height (in)	volume of box (in^3)
0.5	10	7.5	0.5	37.5
1	9	6.5	1	58.5
1.5	8	5.5	1.5	66
2	7	4.5	2	63
2.5	6	3.5	2.5	52.5
3	5	2.5	3	37.5
3.5	4	1.5	3.5	21
4	3	0.5	4	6

Activity Synthesis

The goal of this discussion is to make sure students understand how the volume of the box can be calculated without measuring each dimension, in preparation for writing an equation for the volume in the following activity.

Begin by inviting students to describe what different points mean in this situation. (Each point represents the relationship between the volume of a box and the side length of the square cut out from each corner. The point with the greatest output represents the box with the greatest volume.)

Select previously identified students to share how they calculated the volume of their box without having to measure directly. Then ask students to predict the side length of a cutout that will result in an open-top box with the greatest possible volume. Students may predict side lengths between 1 and 2 inches. Have students use the non-measuring method to calculate the volume of at least one additional side length.

Ask students to keep their boxes out as a visual aid for the remainder of the lesson.